Reply to "Comment on 'Thermodynamic transitions in inhomogeneous *d*-wave superconductors' "

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We discuss the comparison of our theoretical simulations of the specific-heat transition in inhomogeneous cuprate superconductors in the light of the critique of Tallon and Loram [preceding paper, Phys. Rev. B **79**, 096501 (2009)]. We argue that a significant part of the observed transition width may be due to a spatial distribution of the pair interaction, and that the mean field and fluctuation contributions to the specific heat are affected in different ways by inhomogeneity, leaving open the possibility that a proper treatment of fluctuations will give the narrow region of negative curvature observed in experiment.

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In Ref. 1, we reported calculations of the specific heat of a *d*-wave superconductor within a BCS model with spatially inhomogeneous pairing interaction[2,3] developed to understand the inhomogeneity observed in scanning tunneling spectroscopy (STS) gap maps on the surface of $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO).[4] We showed that such an inhomogeneous mean-field approach leads to the creation of superconducting islands when regions with smaller pair interactions become normal, which successively disappear as the temperature T is increased. This picture has indeed been verified by recent high-temperature STS experiments on overdoped samples.[5] What STS cannot verify is whether or not the significant $\pm 25\%$ is a bulk phenomenon or characteristic of the surface layer measured; this was the reason we undertook to connect the STS measurements with bulk properties.

Tallon and Loram,[6] who earlier argued that bulk inhomogeneity in BSCCO is ruled out by their interpretation of the specific-heat data, criticized our analysis on several grounds. We welcome the opportunity to discuss these points here, as we believe the question is of considerable importance to understanding whether the fascinating conclusions drawn from the large body of STS (and angle-resolved photoemission) work is applicable to the bulk BSCCO system or not. We emphasize from the outset that our comments bear specifically on our belief that gross nanoscale electronic inhomogeneity is a bulk property of BSCCO samples as currently prepared, and probably of all "intrinsically doped" cuprates; we do not assert that this inhomogeneity is an essential ingredient of high-T_c superconductivity. For example, the oxygen ordered "ortho-α" crystals of $YBa_2Cu_3O_{7-\delta}$ (YBCO) appear to be extremely homogeneous.

Part of the disagreement with Tallon and Loram is semantic in origin. We refer to the width of the specific-heat transition as the broadening of our simulated specific-heat transition on the high-temperature side of the sharp mean-field transition characteristic of the model with homogeneous BCS coupling constant g equal to its mean value. In Fig. 1 we show the experimental data from Ref. 6, together with Fig. 4 of Ref. 1. Note the qualitatively different behavior of the high-T part of the specific-heat curve between BSCCO and YBCO shown in Fig. 1(a). The width in Fig. 1(b), indicated crudely by a double-headed arrow, depends on the standard deviation in the distribution of coupling constants δg assumed. We show various possibilities in the figure which are semiquantititively consistent with the STS results at low temperatures. In particular, the result for $\delta g = t$ shows a high-temperature broadening of roughly 5 K from the 90 K homogeneous mean-field transition to the point where ΔC has fallen to half its peak value and is even a bit smaller if one subtracts the width of the homogeneous finite-size transition. In this sense the assertion of Tallon and Loram that our results simply reflect the 25% variation assumed in the coupling constant or (calculated) low-temperature order-parameter distribution is incorrect. In fact, the ratio of the width to T_c is closer to 5%, as shown.

Semantics aside, it is clear from examining the two fig-

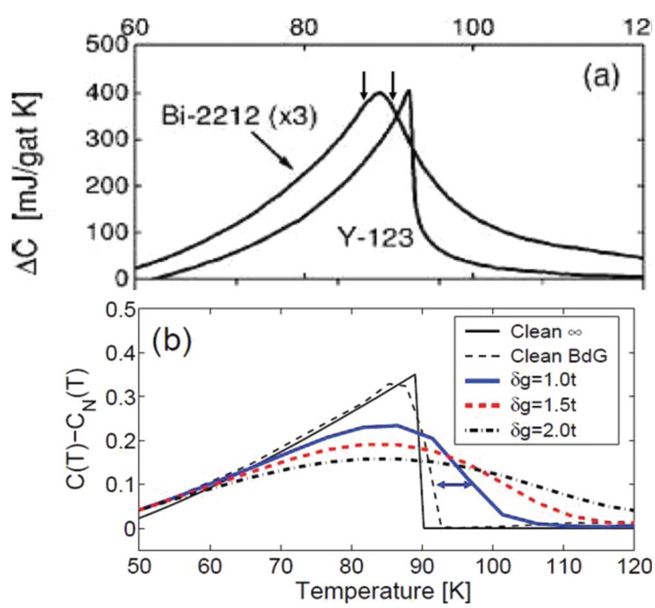

FIG. 1. (Color online) Comparison of phenomenological theory of *d*-wave superconductor with inhomogeneous pair interaction with specific-heat data. (a) Experimental specific heat jump ΔC from Ref. 6 for optimally doped BSCCO and YBCO. (b) Same quantity from theory of Ref. 1 for various values of the modulation of the pair interaction δg.

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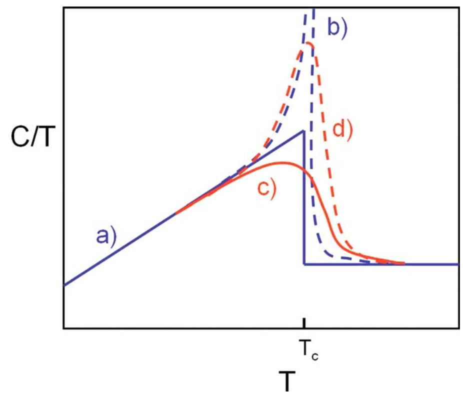

FIG. 2. (Color online)Schematic figure of various contributions to specific heat C/T versus temperature T. (a) Mean-field theory for homogeneous superconductor within BCS theory; (b) homogeneous system including fluctuations, e.g., for 3D*XY* model with small critical exponent $\alpha < 0$; (c) mean-field theory including disorder in pairing channel; (d) full result for inhomogeneous system including fluctuations and inhomogeneity.

ures that the narrow peak of full width 3 K [arrows in Fig. 1(a)], as defined by Tallon and Loram to correspond to the small region of negative curvature near the transition peak, is quite different from the broader peak obtained in our simulation. We speculated in Ref. 1 that the difference between theory and experiment in this regard might be due to our neglect of fluctuations, which would add to the mean-field contribution we have calculated. In the current Comment, as well as Ref. 7, Tallon and Loram argued that this is not expected to occur because the inhomogeneous broadenings of the mean-field and fluctuation contributions to the specific heat are coupled and of the same order. In this context they cite work by Fisher and Barber[8] and Thouless.[9] The former studies the rounding and shift of the transition temperature in thin clean films. It is irrelevant to the present disordered case both because it studies a two-dimensional–three-dimensional (3D) crossover and because the thermodynamic state studied is rigorously zero outside the film; the analogy in the BSCCO system is to a zero-dimensional gap patch whose coupling to its neighbor is *a priori* unknown. The latter work studies critical phenomena in superconductors in the presence of a magnetic field near the upper critical field H_{c2}; this is in a different universality class from our problem. We believe that, in general, there is no simple connection between the broadening induced by inhomogeneity and the broadening induced by fluctuations.

Tallon and Loram claimed that an inhomogeneous model of the type we propose, plus a fluctuation contribution, is unable to reproduce the shape of the observed specific-heat transition with narrow region of negative curvature. To see that this argument cannot *in principle* rule out the effect of fluctuations which we hypothesized in Ref. 1, consider Fig. 2, which shows schematically several contributions to the specific heat of a superconductor near its transition temperature T_c, beginning with the usual BCS result in Fig. 2(a). As is well known, the 3D*XY* fluctuation contribution to the specific heat leads to a quasilogarithmic divergence, as shown in Fig. 2(b).[10] Within a Bogoliubov–de Gennes simulation as in Ref. 1, one can then also imagine the broadened transition which results from adding weak inhomogeneity without fluctuations [Fig. 2(c)]. Now let us imagine adding weak inhomogeneity to the system shown in Fig. 2(b) perturbatively; its effect will be to round and slightly broaden the transition, exactly as stated by Tallon and Loram. This must lead to a situation very similar to that observed in experiment, where a narrow region of negative curvature exists at the transition. The width of the region of negative curvature need not directly reflect the broadening induced by inhomogeneity alone, as seen by comparing Fig. 2(c) and the full result Fig. 2(d). The full result Fig. 2(d) is similar to the inhomogeneous mean-field result with a narrow fluctuation part added, although the additive nature of the two contributions in the general inhomogeneous case is far from obvious. In simulations of granular superconductors,[11] it was shown that the degree of negative curvature near the transition was quite sensitive to the intergranular coupling. In BSCCO the size of this coupling, and indeed whether the system is describable in terms of coupled grains at all, is unknown.

Tallon and Loram also criticized the basis of our model itself for the description of the STS data, pointing out that "the Ni resonance in STS studies show…that superconductivity is weakest and perhaps absent at the points where the supposed gap is maximal." They have failed to understand that our model describes only the local pair field amplitude and is therefore not necessarily linked to phase-coherent pairing. It is thus completely consistent with the fact that the Ni resonances are found only in the large-gap regions, and we have indeed given a quantitative account of this effect in Ref. 12 within the framework of the inhomogeneous gap model.

We close by giving an account of the general experimental situation relevant to the bulk inhomogeneity question in BSCCO. The only experiment capable of directly probing spatial inhomogeneity is STS; this clearly indicates gap patches at the nanoscale[4] but probes only the surface. NMR has some indirect spatial resolution, and indeed O NMR studies have shown that the linewidths of the BSCCO material are several times broader than YBCO and have interpreted this fact in terms of inhomogeneity in doping distributions.[13] This measurement has nothing to do with fluctuations or the superconducting transition. Neutron scattering, also a bulk probe, measures at low temperatures a resonance peak in BSCCO which is several times wider than YBCO but evolves with doping in much the same way, indicating common spin fluctuation physics in the two materials.[14] The authors of this work concluded that this effect was consistent with a bulk distribution of superconducting gaps.[14]

We are grateful to the authors of the Comment for raising the important question of the treatment of fluctuations near the superconducting transition in a strongly inhomogeneous system. In our understanding, this difficult problem is still open. It seems more natural to us at present to assume that the inhomogeneity observed by STS in BSCCO is indeed characteristic of the bulk material, resulting in a broadened high-T part of the specific-heat transition at least qualitatively similar to that we have described.

[1] B. M. Andersen, A. Melikyan, T. S. Nunner, and P. J. Hirschfeld, Phys. Rev. B **74**, 060501(R) (2006).
[2] T. S. Nunner, B. M. Andersen, A. Melikyan, and P. J. Hirschfeld, Phys. Rev. Lett. **95**, 177003 (2005).
[3] T. S. Nunner, P. J. Hirschfeld, B. M. Andersen, A. Melikyan, and K. McElroy, Physica C **460-462**, 446 (2007).
[4] K. McElroy, J. Lee, J. A. Slezak, D. H. Lee, H. Eisaki, S. Uchida, and J. C. Davis, Science **309**, 1048 (2005).
[5] K. K. Gomes, A. N. Pasupathy, A. Pushp, S. Ono, Y. Ando, and A. Yazdani, Nature (London) **447**, 569 (2007).
[6] J. W. Loram, J. L. Tallon, and W. Y. Liang, Phys. Rev. B **69**, 060502(R) (2004).
[7] J. W. Loram and J. L. Tallon, Phys. Rev. B (to be published).
[8] M. E. Fisher and M. N. Barber, Phys. Rev. Lett. **28**, 1516 (1972).
[9] D. J. Thouless, Phys. Rev. Lett. **34**, 946 (1975).
[10] We assume 3D*XY* fluctuations for the sake of argument here. Note that the experimental data have been analyzed both in terms of symmetry classes corresponding to 3D*XY* (Ref. 6) and Bose-Einstein condensate (Ref. 15) behavior.
[11] C. Ebner and D. Stroud, Phys. Rev. B **25**, 5711 (1982).
[12] B. M. Andersen, S. Graser, and P. J. Hirschfeld, Phys. Rev. B **78**, 134502 (2008).
[13] J. Bobroff, H. Alloul, S. Ouazi, P. Mendels, A. Mahajan, N. Blanchard, G. Collin, V. Guillen, and J.-F. Marucco, Phys. Rev. Lett. **89**, 157002 (2002).
[14] L. Capogna, B. Fauque, Y. Sidis, C. Ulrich, P. Bourges, S. Pailhes, A. Ivanov, J. L. Tallon, B. Liang, C. T. Lin, A. I. Rykov, and B. Keimer, Phys. Rev. B **75**, 060502(R) (2007).
[15] A. Junod, A. Erb, and C. Renner, Physica C **317-318**, 333 (1999).